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A NEW CHARACTERIZATION OF THE NYQUIST STABILITY CRITERION,(U)
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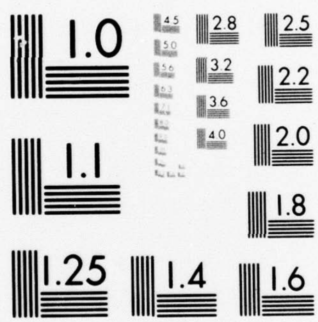
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AUGUST 16-17, 1976

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A NEW CHARACTERIZATION OF THE NYQUIST STABILITY CRITERION*

R. DECARLO & R. SAEKS

ABSTRACT

The usual proof of the Nyquist Theorem depends heavily on the agrument principle. The argument supplies unneeded information in that it counts the number of encirclements of "-1". Stability of a system requires an encirclement or a no-encirclement test. Using homotopy theory, this paper offers a more intuitive approach. We believe this approach will lead to practical generalizations. For example, systems characterized by several complex variables such as multi deminsional digital filters.

I. INTRODUCTION

This paper introduces a characterization of the Nyquist criterion using homotopy theory, a branch of algebraic topology. The authors emphasize the intuition and motivation for this approach. The hope is to aid interested readers to further extend and apply these ideas. In this vein, proofs are omitted so as to simplify the presentation. Details can be found in the references. With this philosophy in mind, let us define the type of system we will be discussing.

As illustrated in Figure 1, let $\hat{g}(s)$ be a rational function in the complex variable s (bounded at $s = \infty$**) representing the open loop gain of scalar single loop feedback system. The closed loop system has transfer function $\hat{h}(s) = \hat{g}(s)/[1+\hat{g}(s)]$. The closed loop system is stable if and only if all poles of $\hat{h}(s)$ are in the open left half plane denoted by $\mathbb{C}_-$ (where $\mathbb{C}$ will denote the entire complex plane).

The Nyquist Criterion states that the closed loop system is stable if and only if the Nyquist plot of $\hat{g}(s)$ (i.e. the image of the Nyquist contour under the map $\hat{g}(\cdot)$) does not encircle nor pass through "-1". If the Nyquist plot passes through "-1" there is a pole on the imaginary axis; if the Nyquist plot encircles "-1", there is a pole in the open right half plane, which we will denote by $\mathbb{C}_+$ ($\bar{\mathbb{C}}_+$ will denote the closed right half plane). The following section constructs the required machinery of homotopy theory

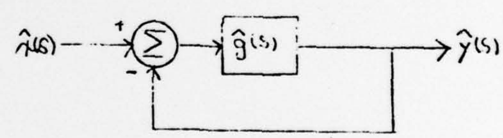
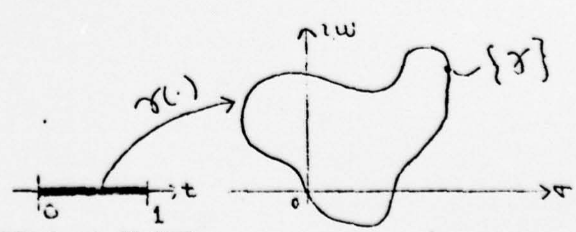


Figure 1

II. MATHEMATICAL PRELIMINARIES & BACKGROUND

Basic to homotopy theory is the concept of a path. A path or a curve in the complex plane is a continuous function of bounded variation (2) $\gamma : [0,1] \to \mathbb{C}$. γ is a closed path if $\gamma(0) = \gamma(1)$. γ is a simple closed path if γ is a closed path and has no self intersections. The image of $I = [0,1]$ under γ is called the trace of γ and is denoted by $\{\gamma\}$.



**This boundedness condition can be dispensed with & is added only to ease the exposition.

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Figure 2

Two closed curves γ_0 and γ_1 are homotopic in $\mathbb{C}$ if there exists a continuous function $\Gamma: I \times I \to \mathbb{C}$ such that:

(a) $\Gamma(s,0) = \gamma_0(s) \quad 0 \leq s \leq 1$

(b) $\Gamma(s,1) = \gamma_1(s) \quad 0 \leq s \leq 1$

(c) $\Gamma(0,t) = \Gamma(1,t) \quad 0 \leq t \leq 1$

Intuitively, γ_0 is homotopic to γ_1 if one can continuously deform γ_0 into γ_1. Moreover, it is easily shown that the homotopy relation is an equivalence relation. (4) (5)

Another important property of a closed curve is its index or degree. The index (2) is of closed curve, with respect to a point "a" not in $\{\gamma\}$ is:

$$n(\gamma;a) = \frac{1}{2\pi i} \int_\gamma (z-a)^{-1}\, dz$$

This integral measures the net increase in angle that the ray r of Figure 3 accumulates as its tip traverses the trace of γ.

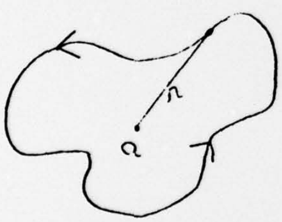

Figure 3

Intuition for the approach stems in part from the observation that $n(\gamma;-1) = 0$ *if and only if* γ is homotopic to a point in $\mathbb{C} - \{-1\}$ (cf. prop. 5.4, ref. 2). We will henceforth refer to such a γ as being homotopically trivial. Conversely, γ encircles "-1" if and only if γ <u>cannot</u> be continuously deformed to a point in $\mathbb{C} - \{-1\}$. These ideas appear to indicate that the Nyquist encirclement condition is fundamentally a homotopy concept.

The intuition is further reenforced when one formulates the Nyquist criterion on the Riemann surface (2) (8) associated with a map, $\hat{f}(s)$. Assuming $\hat{f}(s)$ is analytic on $\mathbb{C}_+$ and bounded at $s=\infty$, the image of simply connected regions in $\mathbb{C}_+$ are simply connected in $\mathbb{C}$. To illustrate the point, let Figure 4-a be the image of the right half plane under $\hat{f}(s)$. The region is not simply connected. Figure 4-b shows the "same region" as it might appear on an appropriate Riemann surface. Here the region is simply connected.

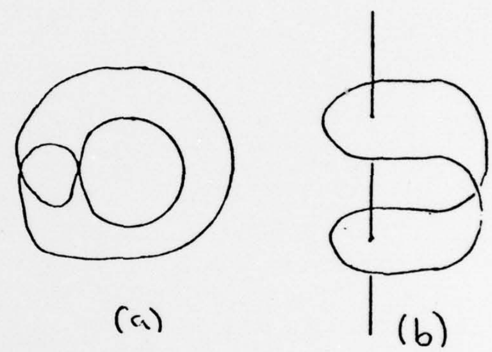

Figure 4

The boundary of the regions depicted in Figure 4 are the Nyquist plots of $\hat{f}(s)$ in $\mathbb{C}$ and on the Riemann surface. On the Riemann surface the Nyquist test becomes an obvious triviality. In $\mathbb{C}$ it is mathematically more delicate.

Our setting uses homotopy theory, a branch of algebraic topology, to establish a topologically invariant relationship between a metric space, X, and an algebraic group called the fundamental group of X, denoted by $\pi(X)$. The relationship is topologically invariant in that homeomorphic spaces have isomorphic fundamental groups.

Specifically, the fundamental group is a set of equivalence classes of closed curves. Each equivalence class consists of a set of curves homotopically equivalent. The group operation is "concatenation" of curves.

For example, the fundamental group of $\mathbb{C}$ consists of one element, $i_{\mathbb{C}}$, the identity, since all closed curves are homotopic to zero. If $X = \mathbb{C} - \{-1\}$, then $\pi(X)$ has a countable number of elements: i_X (the identity) equal to the equivalence class of all closed curves not encircling "-1" and the remaining elements, μ_n ($n = 1, 2, 3 \ldots$) consisting of the equivalence class of all closed curves en-

circling "-1", n times. Moreover, μ_i concatenated with μ_k is equal to the element μ_{k+i}.

Now let X and Y be metric spaces. Let f: $X \to Y$ be locally homeomorphic. In particular, assume that for each point y in Y there exists an open neighborhood G of y such that each connected component of $f^{-1}(G)$ is homeomorphic to G under the map f. Under this condition X is said to be a covering space of Y. (2) (4) Also let $\pi(X)$ and $\pi(Y)$ be the fundamental groups associated with X and Y respectively. With these assumptions, f effects a group isomorphism (i.e. a one to one into mapping preserving group operations) ϕ_f between $\pi(X)$ and a subgroup of $\pi(Y)$ as in the following diagram (4) (5)

$$\begin{tikzcd} X \arrow[r, "F"] \arrow[d, "f"'] & \Pi(X) \arrow[d, "\varphi_f"] \\ Y \arrow[r, "F"] & N \subset \Pi(Y) \end{tikzcd}$$

Figure 5

F is the functor which establishes the relationship between a topological space and its fundamental group. Finally let us distinguish between a critical point and a critical value. A point z_0 in $\mathbb{C}$ is a critical point of a differentiable function f if $f'(z_0) = 0$. A critical value of f is any point $w = f(z_0)$ whenever z_0 is a critical Point.

Now suppose $f: \mathbb{C} \to \mathbb{C}$ is a rational function whose set of poles is $P = \{P_1, \ldots, P_n\}$. Let $Q = \{q_1, \ldots, q_m\}$ be the set of all points in $\mathbb{C}$ such that $f(q_i)$ is a critical value of f. Note that there may be q_i's which are not critical points. To see this consider $g(z) = z^2(z-a)$. $g(0) = 0$ implies "0" is a critical value of g, but $g(a) = 0$ with $g'(a) \neq 0$. Finally, define $T = \{t_i \mid t_i = f^{-1}(-1), i=1,\ldots,n\}$. Note also that since f is a rational function, P, Q and T are finite sets. Define $X = \mathbb{C} - \{P \cup Q \cup T\}$ and define $Y = f(X)$.

Lemma 1: Under the above hypothesis, X is a covering space of Y. This leads to the following corollary.

Corollary: The fundamental group $\Pi(X)$ of X is isomorphic to a subgroup N of $\Pi(Y)$.

This corollary says that a closed curve in X is homotopically trivial.

III. THE SCALAR CASE

Let g(s) be as described in the introduction. Appropriately define the sets P, Q, and T and the spaces X and Y so that X is a covering space of Y. Also as per reference (10) and Figure 6, construct the ugly Nyquist contour, $_R$, and the usual Nyquist contour, Γ, where $\Gamma: I \to \mathbb{C} \cup \{\infty\}$.

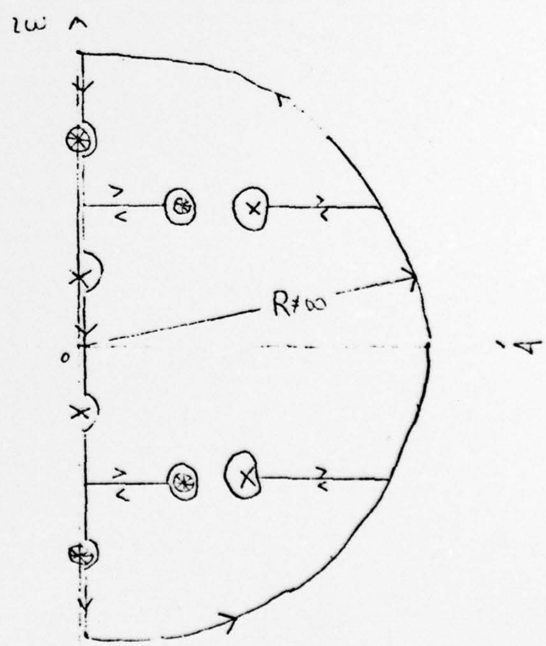

Figure 6-(a)

x indicates a point of P; θ indicates a point of Q

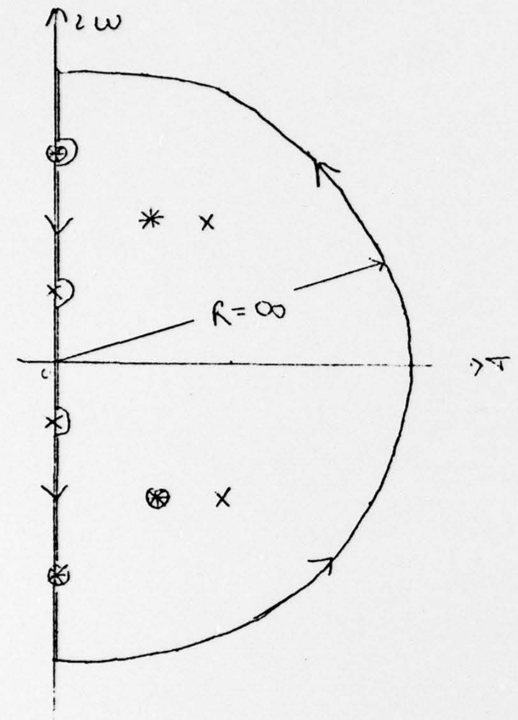

Figure 6-(b)

x indicates a point of P; ⊗ indicates a point of Q

<u>Lemma 2</u>: Under the above assumptions on $\hat{g}$ and λ_R, $\hat{h}(s)$ is stable if and only if the path $\hat{g}o\lambda_R$ does not encircle "-1". (10)

At this point we must establish this lemma's connection with the classical Nyquist criterion. To this end we compare the information of the Nyquist plot, $\hat{g}o\Gamma$ with the "ugly" Nyquist plot, $\hat{g}o\lambda_R$.

<u>Lemma 3</u>: Let n be the number of poles of $\hat{g}$ in $\mathbb{C}_+$, then

$$\frac{1}{2\pi i}\int_{\hat{g}o\Gamma}(z-1)^{-1}dz = \left[\frac{1}{2\pi i}\int_{\hat{g}o\lambda_R}(z-1)^{-1}dz\right] + n$$

These three lemmas give rise to the following theorem.

<u>Theorem 1</u>: Let $\hat{g}(s)$ be as above. Then $\hat{h}(s)$ is stabel if and only if the Nyquist plot of $\hat{g}(s)$ does not pass through "-1" and encircles "-1" exactly n_0 times where n is the number of poles of g(s) in $\mathbb{C}_+$.

IV. MATRIX CASE

Let the entries of an nxn matrix $\hat{G}(s)$ be rational functions in the complex variable s. Suppose $\hat{G}(s)$ characterizes the open loop gain of the single loop feedback system of Figure 7.

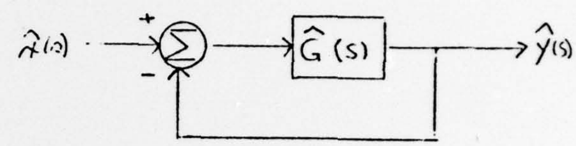

Figure 7

$\hat{x}(s)$ and $\hat{y}(s)$ are n vectors whose entries are also rational functions of s which represent the input and output of the system respectively.

This article assumes each entry of $\hat{G}(s)$ is bounded at x = ∞. Thus $\hat{G}(s)$ as a mapping, $\hat{G}(\cdot):\mathbb{C} \to \mathbb{C}^{nxn}$, is analytic on $\mathbb{C}$ except at a finite number of points, the poles of its entries.

For Figure 7 to be well defined we require that det $[I+\hat{G}(s)]=0$ Thus there exists a closed loop convolution operator, H, such that y = H*x. Moreover the Laplace transform of H, $\hat{H}(s)$ satisfied

$$\hat{H}(s) = \hat{G}(s)[I+\hat{G}(s)]^{-1}$$

For this system to be stable, $\hat{H}(s)$ must have all its poles in $\mathbb{C}_-$ and have all its entries bounded at s = ∞.

Under the assumptions on $\hat{G}(s)$, the following factorization is valid:

$$\hat{G}(s) = N(s)D^{-1}(s)$$

where N(s) and D(s) are right co-prime, polynomial matrices in s with det[D(s)]≠0. Moreover s_0 is a pole of $\hat{G}(s)$ if and only if it is a zero of det [D(s)]. (9)

Desoer and Schulman (3) have shown that the close loop operator H is stable if and only if det[N(s)+D(s)]≠0 for s in $\mathbb{C}_+$ and det[I+G(∞)]≠0. Using this fact, we state and prove the following:

<u>Theorem 2</u>: H is stable if and only if (1) the

Nyquist plot of det[N(s)+D(s)] does not encircle nor pass through "0", and (2) $\det[I+\hat{G}(\infty)]\neq 0$. (10) Observe that if one assumes the open loop gain to be stable (i.e. $\hat{G}(s)$ has all poles in $\mathcal{C}_+$) then det[I+G(s)] in the above theorem. This follows since for all s in $\mathcal{C}_+$, $\det[N(s)+D(s)] = \det[I+\hat{G}(s)]\det[D(s)]$ with $\det[D(s)]\neq 0$. Thus in $\mathcal{C}_+$ det[N(s)+D(s)] has a zero if and only if det[I+G(s)] has a zero.

Finally, it is worthwhile to point out the relationship between the above formulated multivariable Nyquist criterion and that formulated by Barman and Katznelson. For this purpose we let $\lambda_j(iw)$; $j=1, \ldots, n$; denote the n eigenvalues of $\hat{G}(iw)$. In general parameterization of these function by iw is not uniquely determined but one can always formulate such a function. Moreover these functions are piecewise analytic and can be concatonated together in such a way as to form a closed curve which Barman and Katznelson term the Nyquist plot of $\hat{G}(s)$.

Now, since

$$\det[I + \hat{G}(iw)] = \prod_{j=1}^{n} [1 + \lambda_j (iw)]$$

and the degrees of a product is the sum of the degrees of the individual factors and also equals the degree of the concatonation of the factors, the degree of the Barman and Katznelson plot with respect to "-1" coincides with the degree of our plot with respect to "0". As such, even though the two plots are different their degrees coincide and hence either can be used for a stability test.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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